“Mathematics is just 1 + 1 = 2, what is there to argue about?”: Developing a framework for Argument-Based Mathematical Inquiry

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One potential means to develop students’ contextual and conceptual understanding of mathematics is through Inquiry Learning. However, introducing a problem context can distract from mathematical content. Incorporating argumentation practices into Inquiry may address this through providing a stronger reliance on mathematical evidence and reasoning. This paper presents a framework derived from the implementation of multiple, successive Argument-Based Inquiry units to 8-10 year olds. Three key knowledge domains are identified: mathematical, contextual and argumentation knowledge. Key components and roles of each domain are addressed and offered as an initial framework for further research.

> A problem is only a problem if you don’t know how to go about solving it. A problem that has no surprises in store, and can be solved comfortably by routine or familiar procedures (no matter how difficult) is an exercise (Schoenfeld, 1983: p. 41).

Mathematics education reform has been an ongoing goal in many countries for decades, the aim of which is to place a heavier focus on understanding of content and processes in context (NCTM, 2000). One approach which has found some prominence in meeting the goals of reform has been mathematical inquiry. Hmelo-Silver, Duncan, and Chinn (2007) define inquiry learning (IL) as a process in which “*students are cognitively engaged in sense making, developing evidence-based explanations, and communicating their ideas*.” (p. 100). This is achieved by engaging students with the discipline content through collaborative engagement in investigations: investigations which address contextualised, complex, ill-structured problems that have neither a correct answer nor a clearly defined approach (Makar, 2012). Such problems require students to pose a refined question that is both defined and mathematised (Allmond & Makar, 2010), to gather and analyse evidence, and to provide a response. However, contextualisation of mathematics has given rise to some criticism, with one significant concern being the possibility for mathematics and mathematical language to become lost through focus on the context (Wu, 1997). One possibility for supporting the focus on mathematics in IL is the introduction of argumentation.

Argumentation as a pedagogy has demonstrated enhanced student understanding, discourse, and ways of knowing specific to science (Jimenez-Aleixandre & Erduran, 2007). Literature addressing argumentation research in the mathematics domain is readily available; however, much of this research involves argumentation based on mathematical proof (e.g. Conner, 2007; Lampert, 1990) or mathematical procedure (e.g. Goos, 2004; Yackel & Cobb, 1996). In no way does the research here attempt to detract from those approaches, rather, the purpose is to address an additional approach, that is, argumentation applied to problems which are both undefined in terms of solution and solution pathway. Overlaying inquiry with argumentation provides students with the opportunity to engage in these complex problems while a clear expectation of the provision of mathematical evidence and reasoning is established.

One of the potential complexities with introducing argumentation to young students is the lack of familiarity with argumentation practices. Argument essentially exists in two forms; it is both a product and a discursive practice (Leitão, 2000). Blair defines argument

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as “*a set of one or more reasons for doing something*” and argumentation as “*the activity of making or giving arguments*” (Blair, 2012, p. 72). The most widely used model of argumentation is that of Toulmin (1958). However, this model is predominantly a structural approach to argument which, while useful for argument as product, provides no guarantee that any evidence or reasoning presented is epistemically acceptable (Kelly, Druker, & Chen, 1998). McNeill and colleagues provide a Claim-Evidence-Reasoning (CER) model which derives from the more complex Toulmin model of argument but which has been adapted to provide a simplified model for science education purposes (McNeill & Krajcik, 2008, McNeill & Martin, 2011). Another advantage of the CER model is that the claim and evidence components take on an evidenced approach: *claim* being the conclusion that addresses the original question and *evidence* being the scientific data that supports the claim. A third component, *reasoning*, focuses on bringing in the scientific background knowledge or scientific theory that justifies making a claim-to-evidence link.

The CER model used in science education provides a means in mathematics education to focus students on the use of evidence and reasoning appropriate to the discipline when responding to complex inquiry-based problems (Wells, 2014). Accordingly, the aim of the research presented in this paper is to provide a working framework that conveys the essential components of mathematical argument-based inquiry (ABI) structured on the CER model.

Research Context

This research study addressed the use of argumentation to enhance the learning of students who were engaging with IL practices. Due to the pragmatic nature of the research, the desire to create theory, and the likely need to implement multiple iterations of ABI, design-based research was adopted as a methodological approach (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003).

The findings reported here derive from a larger project which took place at a metropolitan government primary school in Australia. In this project, various classes of students, ranging from Foundation to Year 5, who had been engaged in IL were introduced to ABI. The findings reported here derive from analysis of a class of initially Year 4 students over a period of 18 months. The students in this study had become accustomed to addressing complex questions and the classroom culture was one that supported IL (for further detail about Inquiry culture, refer to Makar, 2012). Three iterations (10-15 lessons each) of ABI were conducted with this class and video-recorded in full (see Table 1).

Each unit was viewed and logged lesson by lesson, with time stamps, excerpts of students’ work, and still shots of teaching materials. This was done to illustrate the nature and context of each lesson, provide an overview, and enable cross comparison between the units for particular patterns in the development of the inquiry. Each lesson was transcribed in full by the researcher, and then relevant sections were coded through either theory derived concept names previously identified through the literature (for example, ‘evidence’, ‘claim’, ‘question’) or through participant derived concept names (for example, ‘evidence for the conclusion’) (Corbin & Strauss, 2008). Coding continued until such time as saturation was deemed achieved. These codes were clustered into code categories and substantive categories were developed and used to map themes and relationships (Clarke, 2005, p. 83). From this process, three knowledge domains were identified: mathematical, contextual and argumentation. To illustrate, two responses given by students to support their claims that shower timers were useful, are provided along with the coding chain in parentheses.

Dominica: We tested every shower timer 5 times we worked out that 18/30 were accurate and ran in the range of 3:36 – 4:24. For the other 12 shower timers they were faulty because they were out of the range … [or] they stopped during timing. [Argument Knowledge/Evidence Quality/Mathematical Evidence]

Leanne: …if you have a hot shower you could use up all the hot water. [Argument Knowledge/ Evidence Quality/Irrelevance] [Context Knowledge/Prior Experience/Impeding Evidence]

This provided the skeleton of the model and also some insight into the roles of both the domains and the components of the domains by examining the material coded to each component. As a result, a model of ABI was developed, along with some insight into the components of these domains (for detailed explanation, refer Wells, 2014).

Table 1
Overview of the ABI unit sequence

	Inquiry Question (Context)	Mathematical Content Addressed	Argumentation Structure Addressed	Argumentation Processes Addressed
1	Does Barbie have the same proportions as a human? Term 4, Year 4	Proportional reasoning Informal representation Fractional representations Informal inference Distribution Samples vs populations Data representations – tallies, dot plots	Informal introduction of Claim – Evidence Role of Evidence	Informal introduction of Claim – Evidence links Challenging evidence
2	Can a pyramid have a scalene face? Term 3, Year 5	Geometrical reasoning Properties of triangles Properties of pyramids Angles	Formal introduction of claim, evidence, reasoning and qualification Quality of evidence Scaffolded argument	Envisaging and gathering evidence Selecting evidence for inclusion Mathematical Reasoning
3	Are government issued shower timers accurate? Term 4, Year 5	Time: duration & measurement (min:sec) Data recording and representation Measures of centre Variability	Quality of evidence Construction of arguments with limited scaffolding Consideration of qualifiers and rebuttals	Critical examination of arguments. Planning collection, representation and analysis of data Statistical Reasoning Articulating claims

Theoretical Model

The ABI model (Figure 1) incorporates three knowledge domains: the mathematical knowledge required to progress through the problem; the understandings that surrounded the context employed; and, the knowledge of structures and conventions of argumentation. At any one time, students could be drawing on one or more of these domains to engage with ABI. This section will provide an overview of each domain followed by a discussion on the role of each domain in ABI.

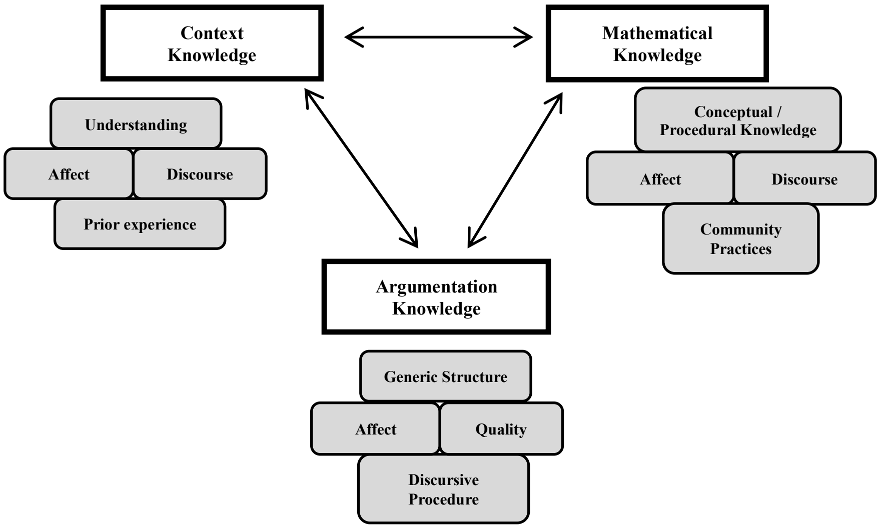

Figure 1. Model of interacting domains in Argument-Based Inquiry.

Context knowledge

Four key components of Context Knowledge were identified: *prior experiences*, *understanding*, *affect*, and *discourse*. *Prior experience* with the context describes the past engagement that the students had with the problem context. These experiences may be quite different for each student, they may be school or family based, culturally specific, or gender based. This is closely related to the student's *understanding* of the context as this may derive from experience, but may also derive from teaching or reading for example. Prior experiences are considered in this instance as those occurrences that are not open to change as the experiences are historically situated – they have occurred; however, understandings, even if built on prior experiences, are able to be challenged, deepened and altered through subsequent experience or knowledge. *Affect* also takes a part in context knowledge as prior or ongoing experiences and understandings may instil and elicit emotive responses in students that are associated with the context. Students may hold particularly strong feelings or intuitively held beliefs about a context and these may serve to influence their engagement or interpretation of the results. The final component identified was the *discourse* of the context. Students' familiarity with the context has the potential to provide an underlying language and terminology for use. While this is not essential to the mathematical understanding, it has the potential to support or challenge students' involvement in classroom discourse.

Context knowledge serves to situate the application of mathematics, authenticate the learning, and engage students in the learning sequence. For example, the first learning sequence enabled connections between aspects of mathematics (fractions, proportion, and statistics), other curriculum areas (human proportion in the Arts) and the 'real' world (clothing manufacture). Context also served as a 'hook' to engage students in the mathematical learning. In the same unit, students spent some time talking about the unit purpose and reflecting on reasons why 'normal' proportions for humans would be required, including forensic applications. Students were thus able to see important reasons for establishing human proportions that were connected to the real world.

Argumentation Knowledge

The second domain of the proposed model is *Argumentation Knowledge,* which draws upon the knowledge the students have around argument structure and argument process. Analysis of the classroom activities and interactions, along with consideration of prior literature in this area, suggests several components of Argumentation Knowledge significant to ABI: *generic structure* of the argument, *quality* of the argument and *discursive process*.

During the second unit, the students negotiated and developed their own model of ABI which they felt was useful in supporting the process for others (Figure 2). It shows the IL focus of Purpose-Question-Evidence-Conclusion (refer to Fielding-Wells, 2010) while expanding the Conclusion to encompass the generic structure of the CER model (McNeill & Krajcik, 2008; McNeill & Martin, 2011). One additional category, qualifiers, was introduced to meet student need. Analysis showed students using qualifiers in two distinct ways: as modal qualifiers and as 'delineating qualifiers'. Modal qualifiers were used to express the strength of a claim - probably, certainly, possibly - and appeared most frequently in arguments involving informal statistics to express a level of uncertainty about inferences made from sample data (see informal statistical inference, Makar & Rubin, 2009). The second type of qualifiers served to identify limits to the conditions that the claim applied to. The nature of ill-structured questions is such that the questions needed refining and negotiating to researchable point and this is of itself limiting. Thus, qualifiers may provide a way of expressing the limitations surrounding the inquiry and impacting on the argument: For example: For the measurement foot length to lower arm length, Barbie's proportions fell outside the range of a normal human.

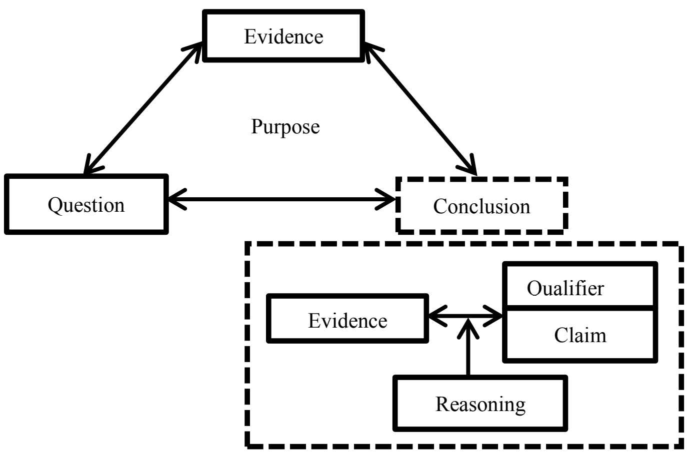

Figure 2: Process of Argument-Based Inquiry.

The second component of Argumentation Knowledge is *discursive process*. The mathematics argument is essentially a discursive practice, even if one is arguing with oneself. However, it is not a practice which is usually encountered in the learning environment, and hence a culture of argument and the practice of "appropriate arguing" needs to be developed in the classroom (Pontecorvo & Pirchio, 2000).

Argument *quality* is the third component to Argument Knowledge. Stressing the importance of quality enables students to see the importance of the role of evidence, both in helping to support a strong claim and it making such a claim in the first place. Affect, the final component, is important in argumentation practice as the students need to be willing to engage in argumentation activities as there is potential for this to be confronting.

The overall role of argumentation knowledge is to provide students with a support or structure to frame their own argument products and present their arguments as a process. Having students construct and present their arguments in written and oral format serves to organise and demonstrate their thinking. As students or student groups articulate their claim, evidence and reasoning, there is valuable opportunity for students and teachers to probe understanding and thinking quite deeply.

Mathematical Knowledge

The proposed model of ABI also incorporates *Mathematical Knowledge*. Enhanced *conceptual and procedural knowledge* is a key component and was a primary reason for the implementing of ABI based prior success in science education research, as demonstrated by Jimenez-Aleixandre and Erduran (2007). Findings from the mathematics research addressed in this paper suggested that was the case for mathematics also as students' arguments over the course of the research showed gains in evidence quality across multiple indicators; including, evidence collection, organisation, representation, interpretation and reasoning (Wells, 2014).

A second component of Mathematical Knowledge reflects the nature of *community practice*. As students develop in knowledge, their mathematical practices need to increasingly approximate the authentic practices of mathematicians (Collins, Brown, & Newman, 1989). By taking part in the community practice of mathematics, students develop understanding of what is important and valued by the mathematical community. At the commencement of the initial unit, only two students selected data as essential in convincing others of the accuracy of their claim. By the conclusion of the second unit, all students were providing objective data and, in the third unit, were examining evidence critically.

Discourse is an important component of building mathematical knowledge. Through engaging in discussion, students have the opportunity to express ideas, have them challenged and thus develop robust reasoning. They also have opportunities to use the language of mathematics and this helped them develop their understanding of concepts and terms within context, giving students the opportunity to express themselves precisely. The opportunity to use these terms and develop contextualized meaning is vital to students' engagement in mathematical practice (Lee & Herner-Patnode, 2007).

Affect was not a specific focus of this research but warrants inclusion in this model and is flagged as an area of future research. Mathematics is a subject area that has been characterized by poor student engagement (McPhan, Moroney, Pegg, Cooksey, & Lynch, 2008). Early research reported elsewhere (Fielding-Wells & Makar, 2008) suggests that IL has potential to curb and even reverse aspects of disengagement in mathematics among primary aged children and hence, flagging this for further research is important.

Mathematical Knowledge plays an important role in ABI for the obvious reason that a student cannot argue mathematically without drawing on mathematics. However, the need to address a question using mathematically-derived evidence requires that students can envisage the evidence, and thus the mathematical concepts and procedures that would be useful in addressing the question. Students demonstrated on multiple occasions that, even if they didn't have the mathematics needed, they could envisage what they needed to be able to do and request instruction. For example, in the first unit, students could envisage what needed to be done – finding a means of comparing a Barbie doll to a human – but the difficulty arose in that the students did not know how to do that. This quandary gave students their first insight into a mathematical need for proportional reasoning and they requested that the teacher show them a mathematical way to make such comparisons.

Conclusions and implications

The research goal was to identify some key features of an ABI model as implemented in a primary mathematics setting. Three domains necessary to the development of ABI have been identified as Mathematical Knowledge, Argumentation Knowledge and Context Knowledge and each has a significant role to play in learning through ABI. Brown and Campione (1996) suggest that a framework such as that provided here should "contribute to a theory of learning that can capture and convey the essential features of the learning environments that we design". The intent has been to contribute to such a theory, not as an accomplishment, but rather as an early framework that will flag the potential of ABI in mathematics learning and provoke discussion and research into the area. By conceptualising ABI, researchers and educators have an increased opportunity to visualize what it entails, and be cognisant of components for planning and implementation purposes. Development of a model also potentially promotes a common language around which to articulate discussion and critique of ABI.

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